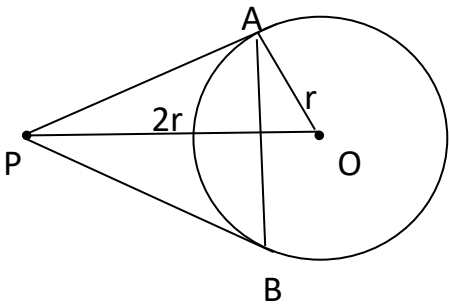**Time Allowed: 2 hours**   **Maximum Marks: 40**
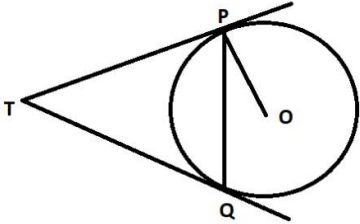
General Instructions:
1.   The question paper consists of 14 questions divided into 3 sections A, B, C.
2.   All questions are compulsory.
3.   Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4.   Section B comprises of 4questions of 3 marks each. Internal choice has been provided in one question.
5.   Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

	Section A	
Q No		**Marks**
1	Find the value of a_{25} - a_{15} for the AP: 6, 9, 12, 15, ……….. OR If 7 times the seventh term of the AP is equal to 5 times the fifth term, then find the value of its 12th term.	**2**
2	Find the value of m so that the quadratic equation $mx(5x-6) = 0$ has two equal roots.	**2**
3	From a point P, two tangents PA and PB are drawn to a circle C(0, r). If OP = 2r, then find $\angle APB$. What type of triangle is APB? 	**2**
4	The curved surface area of a right circular cone is 12320 cm². If the radius of its base is 56cm, then find its height.	**2**
5	Mrs. Garg recorded the marks obtained by her students in the following table. She calculated the modal marks of the students of the class as 45. While printing the data, a blank was left. Find the missing frequency in the table given below . 	Marks Obtained	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	 |---|---|---|---|---|---|	**2**

	Number of Students	5	10	---	6	3	

6	If Ritu were younger by 5 years than what she really is, then the square of her age would have been 11 more than five times her present age. What is her present age? **OR** Solve for x: $9x^2 - 6px + (p^2 - q^2) = 0$	2

Section-B

7	Following is the distribution of the long jump competition in which 250 students participated. Find the median distance jumped by the students. Interpret the median	3

Distance (in m)	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5
Number of Students	40	80	62	38	30

8	Construct a pair of tangents to a circle of radius 4cm, which are inclined to each other at an angle of 60°.	3

9	The distribution given below shows the runs scored by batsmen in one-day cricket matches. Find the mean number of runs.	3

Runs scored	0 - 40	40 - 80	80 - 120	120 - 160	160 - 200
Number of batsmen	12	20	35	30	23

10	Two vertical poles of different heights are standing 20m away from each other on the level ground. The angle of elevation of the top of the first pole from the foot of the second pole is 60° and angle of elevation of the top of the second pole from the foot of the first pole is 30°. Find the difference between the heights of two poles. (Take √3 = 1.73) **OR** A boy 1.7 m tall is standing on a horizontal ground, 50 m away from a building. The angle of elevation of the top of the building from his eye is 60°. Calculate the height of the building. (Take √3 = 1.73)	3

Section-C

11	The internal and external radii of a spherical shell are 3cm and 5cm respectively. It is melted and recast into a solid cylinder of diameter 14cm, find the height of the cylinder. Also find the total surface area of the cylinder. (Take $\pi = \frac{22}{7}$)	4

12	Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact to the centre.	4
	OR	
	Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$	
13	**Case Study-1**	
	Trigonometry in the form of triangulation forms the basis of navigation, whether it is by land, sea or air. GPS a radio navigation system helps to locate our position on earth with the help of satellites.	
	A guard, stationed at the top of a 240m tower, observed an unidentified boat coming towards it. A clinometer or inclinometer is an instrument used for measuring angles or slopes(tilt). The guard used the clinometer to measure the angle of depression of the boat coming towards the lighthouse and found it to be 30°.	
	(Lighthouse of Mumbai Harbour. Picture credits - Times of India Travel)	
	i) Make a labelled figure on the basis of the given information and calculate the distance of the boat from the foot of the observation tower.	2
	ii) After 10 minutes, the guard observed that the boat was approaching the tower and its distance from tower is reduced by 240(√3 - 1) m. He immediately raised the alarm. What was the new angle of depression of the boat from the top of the observation tower?	2
14	**Case Study-2**	
	Push-ups are a fast and effective exercise for building strength. These are helpful in almost all sports including athletics. While the push-up primarily targets the muscles of the chest, arms, and shoulders, support required from other muscles helps in toning up the whole body.	

Nitesh wants to participate in the push-up challenge. He can currently make 3000 push-ups in one hour. But he wants to achieve a target of 3900 push-ups in 1 hour for which he practices regularly. With each day of practice, he is able to make 5 more push-ups in one hour as compared to the previous day. If on first day of practice he makes 3000 push-ups and continues to practice regularly till his target is achieved. Keeping the above situation in mind answer the following questions:
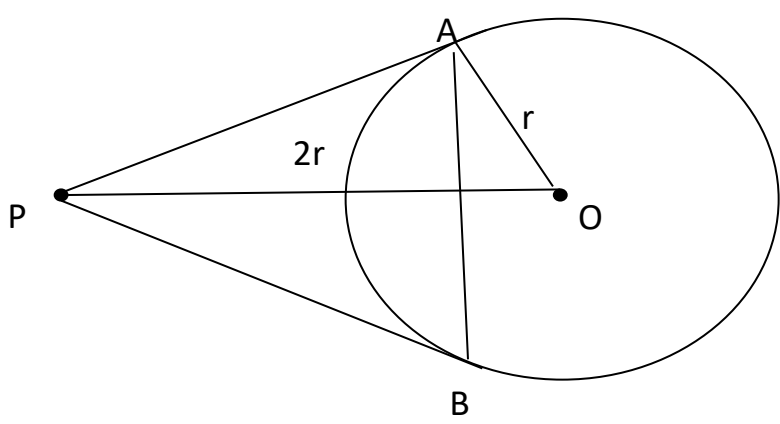
i) Form an A.P representing the number of push-ups per day and hence find the minimum number of days he needs to practice before the day his goal is accomplished?

ii) Find the total number of push-ups performed by Nitesh up to the day his goal is achieved.

2

2

Marking Scheme
Class- X, Session- 2021-22
TERM II
Subject- Mathematics (Standard)

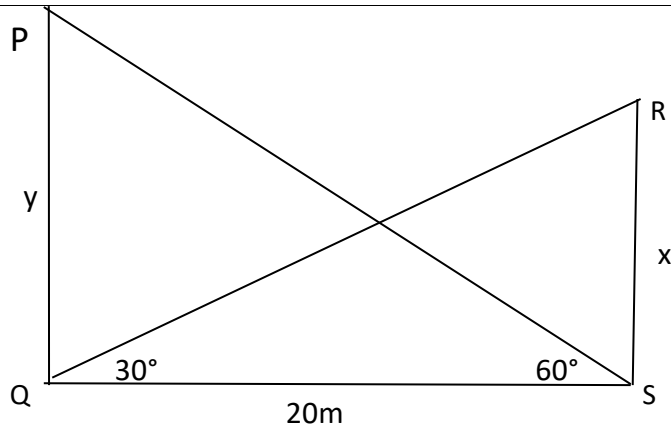| Q.No.	SECTION A HINTS/SOLUTION	MARKS
1	a = 6, d = 3 ; a_{25} = 6 + 24(3) = 78 a_{15} = 6 + 14(3) = 48 ; $a_{25} - a_{15}$ = 78 - 48 = 30 **OR** $7(a + 6d) = 5(a + 4d)$ $\Rightarrow 2a + 22d = 0 \Rightarrow a + 11d = 0 \Rightarrow t_{12} = 0$	1 1 1 1
2	5mx² - 6mx + 9 = 0 b² - 4ac = 0 $\Rightarrow$ (-6m)² - 4(5m)(9) = 0 $\Rightarrow$ 36m(m - 5) = 0 $\Rightarrow$ m = 0, 5 ; rejecting m=0, we get m = 5	 1 1
3	 let $\angle APO = \theta$ $Sin\theta = \dfrac{OA}{OP} = \dfrac{1}{2} \Rightarrow \theta = 30^0$ $\Rightarrow \angle APB = 2\theta = 60^0$ Also $\angle PAB = \angle PBA = 60^0 \ (\because PA = PB)$ $\Rightarrow \triangle APB$ is equilateral	 1/2 1/2 1/2 1/2
4	CSA (cone) = $\pi r l = 12320$ $\dfrac{22}{7} \times 56 \times l = 12320$ $l = 70 \ cm$ $h = \sqrt{70^2 - 56^2} = 42$ cm	1/2 1 1/2

5	Modal class is $40-60, l=40, h=20, f_1=?, f_0=10, f_2=6$	1/2
	$45 = 40 + 20 \times \left[\dfrac{f_1 - 10}{2f_1 - 10 - 6}\right]$	1/2
	$\Rightarrow \dfrac{1}{4} = \dfrac{f_1 - 10}{2f_1 - 16}$	
	$\Rightarrow 2f_1 - 16 = 4f_1 - 40 \Rightarrow f_1 = 12$	1
6	Let the present age of Ritu be x years	
	$(x - 5)^2 = 5x + 11$	1
	$x^2 - 15x + 14 = 0$½	
	$(x - 14)(x - 1) = 0 \Rightarrow x = 1 \ or \ 14$	1/2
	x = 14 years (rejecting $x = 1$ as in that case Ritu's age 5 years ago will be –ve)	1/2
	OR	
	$9x^2 - 6px + (p^2 - q^2) = 0$	
	$a = 9, \quad b = -6p, \quad c = p^2 - q^2$	1/2
	$D = b^2 - 4ac = (-6p)^2 - 4(9)(p^2 - q^2) = 36q^2$	1/2
	$x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{6p \pm 6q}{18} = \dfrac{p+q}{3} or \dfrac{p-q}{3}$	1

<div align="center">**SECTION B**</div>

7	Distance (in m)	0 - 1	1 – 2	2 - 3	3 - 4	4 - 5	
	Number of Students	40	80	62	38	30	
	cf	40	120	182	220	250	

$\dfrac{n}{2} = \dfrac{250}{2} = 125 \Rightarrow median \ class \ is \ 2-3, l=2, h=1, cf=120, f=62$ **1**

$median = l + \dfrac{\dfrac{n}{2} - cf}{f} \times i$ **1/2**

$= 2 + \dfrac{5}{62}$

$= \dfrac{129}{62} = 2\dfrac{5}{62} m \ or \ 2.08m$ **1**

50% of students jumped below $2\dfrac{5}{62}$ m and 50% above it.   **1/2**

8	Draw a circle of radius 4cm	1
	Draw OA and construct $\angle AOB = 120^0$	1
	Draw $\angle OAP = \angle OBP = 90^0$	
	PA and PB are required tangents	1

9	Runs Scored	0 - 40	40 - 80	80 - 120	120 - 160	160 - 200	TOTAL
	Number of Batsmen (f_i)	12	20	35	30	23	120

x_i	20	60	100	140	180		
$f_i x_i$	240	1200	3500	4200	4140	13280	$1\frac{1}{2}$

$$mean\ (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i} = \frac{13280}{120} = 110.67\ runs \qquad 1\frac{1}{2}$$

10

$$In\ \Delta PQS,\ \tan 60^0 = \frac{y}{20} \Rightarrow y = 20\sqrt{3}m \qquad 1/2$$

$$In\ \Delta RSQ, \qquad \tan 30^0 = \frac{x}{20} \Rightarrow x = \frac{20}{\sqrt{3}}m \qquad 1/2$$

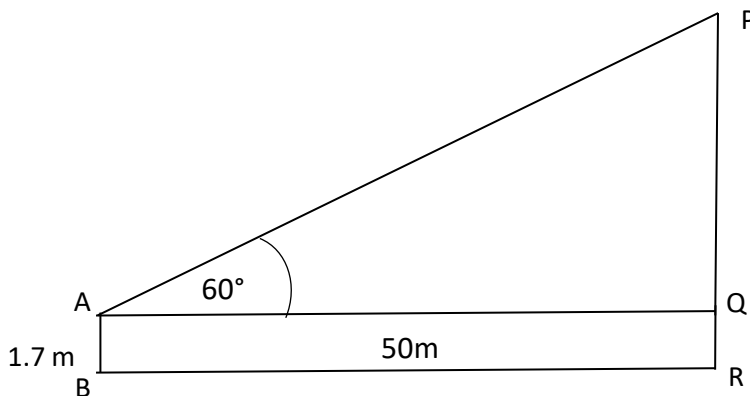$$y - x = 20\sqrt{3} - \frac{20}{\sqrt{3}} = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3} = 23.06m \qquad 1$$

OR

Let PR be the building and AB be the boy

$$In\ \Delta PQR, \tan 60^0 = \frac{PQ}{50} \Rightarrow PQ = 50\sqrt{3}m \qquad 1$$

Height of the building = $PR = (50\sqrt{3} + 1.7)m = 88.2m$ 1
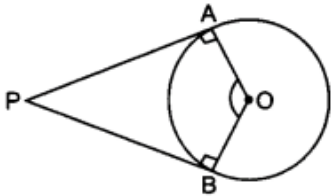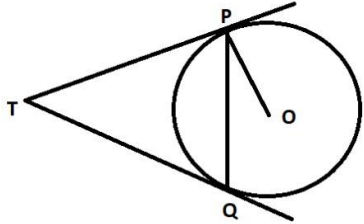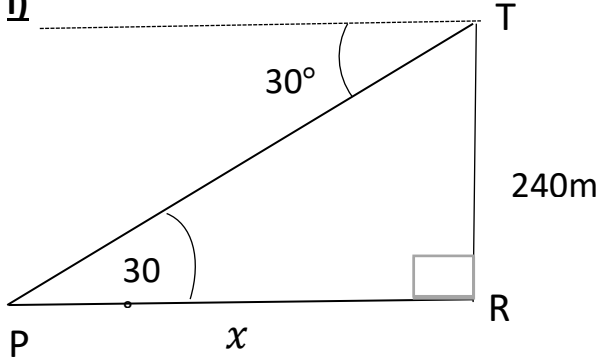
	SECTION C	
11	Volume of shell = Volume of cylinder	
	$\Rightarrow \frac{4\pi}{3}[\,5^3 - 3^3] = \pi(7)^2 h$	$1\frac{1}{2}$
	$\Rightarrow h = \frac{8}{3} = 2\frac{2}{3}cm$	1

	TSA of cylinder is $= 2\pi r(r + h) = 2 \times \frac{22}{7} \times 7 \times \left(7 + \frac{8}{3}\right) = 44 \times \frac{29}{3} = \frac{1276}{3}cm^2 \ or \ 425.33cm^2$	$1\frac{1}{2}$
12	 $\angle OAP + \angle OBP + \angle APB + \angle AOB = 360^o$ $\Rightarrow 90^0 + 90^0 + \angle APB + \angle AOB = 360^o \ (\because Tangent \perp radius)$ $\Rightarrow \angle APB + \angle AOB = 180^0$ **OR** Let $\angle PTQ = \theta$ TPQ is an isosceles triangle. $\angle TPQ = \angle TQP = \frac{1}{2}(180^o - \theta) = 90° - \frac{\theta}{2}$ $\angle OPT = 90^o$ $\angle OPQ = \angle OPT - \angle TPQ = 90^o - \left(90^o - \frac{\theta}{2}\right) = \frac{\theta}{2}$ $\angle OPQ = \frac{1}{2}\angle PTQ$ $2\angle OPQ = \angle PTQ$	1 $1\frac{1}{2}$ $1\frac{1}{2}$ $1\frac{1}{2}$ $1\frac{1}{2}$ 1
13	**Case Study-1** **i)** In ΔPTR, $\tan 30^o = \frac{240}{x} \Rightarrow x = 240\sqrt{3}\ m$	1 1

	ii) Distance of boat from tower$= 240\sqrt{3} - 240(\sqrt{3} - 1) = 240m$ Let the angle of depression = θ $tan\theta = \frac{240}{240} = 1 \Rightarrow \theta = 45^o$	1 1

14	i) 3000, 3005, 3010, …,3900.
	          $a_n = a + (n - 1)d$
	       3900 = 3000 + (n - 1)5
	      $\Rightarrow 900 = 5n - 5 \Rightarrow 5n = 905 \Rightarrow n = 181$
	 Minimum number of days of practice = $n - 1 = 180\ days$
	
	 ii) $S_n = \frac{n}{2}(a + l)$
	           $= \frac{181}{2} \times (3000 + 3900) = 624450$ pushups
		1
		
		 1
		
		 1
		
		 1